# Fast Design of Unconditionally Stable Power Amplifier Using the Center Frequency Smith Tube

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Abstract — Unconditional stability is often a desired criterion in amplifier design. The Smith Tube, a threedimensional cylindrical extension of the Smith Chart with center frequency represented on the height axis, can be used as a tool to design an unconditionally stable amplifier. The stability circles taken over frequency are extended into a stability surface in the Smith Tube representing regions of stability and instability across a range of frequencies. This aids the designer in selecting parameters that will result in an unconditionally stable amplifier design over all frequencies. While this paper discusses stabilization using the Smith Tube, the Frequency Smith Tube can also be used to assess gain and noise performance over frequency, and to plot the reflection coefficient presented by a matching network to a device as frequency is varied. This allows visualization of a bandwidth-based design in a single view.

*Index Terms* — amplifier design, unconditional stability, Smith tube

#### I. INTRODUCTION

Amplifiers are typically designed to be stable (in a first design pass) by analyzing the stability circle at the design frequency of the amplifier and adjusting the circuit until the stability circle has moved completely out of the Smith Chart. The stability circle and gain circles can be plotted using the small signal S-parameters as described in [1]. Once the designer has plotted these circles on the Smith Chart, the input and output matching networks of the amplifier can be designed following the process outlined inCosopo [2]. After design, the stability of the circuit can be analyzed by measuring the geometrically derived stability factors [3]. In addition, Suarez presents stability problems that arise in the large-signal regime including spurious oscillations and frequency divisions [4]. Other methods can be employed in amplifier design based on large signal stability analysis including calculation of system poles and zeroes and using Floquet theory [5,6]. However, the small signal stability analysis is a necessary starting point, is commonly used, and allows for use of the powerful visual tool, the Smith Chart.

Unfortunately, the Smith Chart stability circle design method only identifies instability at the design frequencyuseful, and ignores the fact that the amplifier could be unstable outside of the design frequency. A design tool is needed that can display the stability of the amplifier over a span of frequencies. Fellows introduces the concept of a three-dimensional Smith Chart with bandwidth as the third dimension, also known as a Smith Tube [7]. A three dimensional extension of the Smith Chart has also been developed for optimization of circuit input power and transistor bias simultaneously with load impedance [8,9].

A similar Smith Tube, using frequency as the third dimension, enables a designer to analyze a stability surface over a span of frequencies. If the stability surface remains outside the Smith tube for all frequencies (in the case where  $|S_{11}| < 1$  and  $|S_{22}| < 1$ ), then it is certain that the amplifier will be unconditionally stable. In this paper, the original design method that analyzes stability at only the center frequency will be compared with the proposed design method that uses the Frequency Smith Tube to ensure stability.

For the designs compared, the simulation used a built-in NPN BJT model in Keysight Technologies' Advanced Design System (ADS). This transistor was biased at  $V_{CE} = 4.95$  V and  $I_{BB} = 100 \mu$ A. The design frequency was 1 GHz. The gain of the amplifier at the design frequency was about 16 dB for both designs. The difference between the two designs is the value of the shunt resistor added at the output to improve stability.

## II. THE CENTER FREQUENCY SMITH TUBE

Fellows presents the Smith Tube as a tool for simultaneous optimization of several circuit parameters in [6,7,8]. A similar Smith Tube can be used as a tool to achieve an amplifier design that is unconditionally stable. This Smith Tube is shown in Figure 1. The z-axis of this Smith tube represents center frequency. Just as stability circles can be plotted on the two-dimensional Smith chart to illustrate impedances that lie in the potentially unstable region, a three dimensional stability surface can be plotted in the Smith Tube to show load impedances across a range of frequencies that result in potential instability. The benefit of the visualization of regions of instability in three dimensions is that instability often occurs at a frequency lower than the design frequency of the amplifier. The Smith Tube stability surface allows the designer a more complete picture of the parameters

needed to achieve an amplifier that is unconditionally stable at all frequencies.



Fig. 1. The Smith tube. The vertical axis represents the center frequency, while the horizontal cross section of the tube is a conventional Smith Chart.

## **III. SMITH CHART DESIGN METHOD RESULTS**

In a typical amplifier design process, one of the first steps taken is to plot the linear stability circles of the device at the design frequency using the device Sparameters. If the stability circles intersect the Smith chart, the device is potentially unstable. Thus the designer can choose a new transistor bias or add resistors at the input or output to move the stability circles outside of the Smith chart. Figure 2 depicts the stability circle for the device at the design frequency of 1 GHz and chosen bias point. As seen in the figure, the stability circle lies outside of, but close to the Smith chart. Since there are always slight differences between simulated and fabricated circuits, in this case, the designer would likely choose to add a shunt resistor at the output in order to move the stability circle even farther away from the Smith chart. Figure 3 shows the location of the stability circle for the same device with a 600  $\Omega$  shunt resistor at the output of the transistor.

$$K = \frac{(1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2)}{2|S_{12}S_{21}|},$$
 (1)

where

$$\Delta = S_{11}S_{22} - S_{12}S_{21}, \tag{2}$$

as described by Gonzalez [1]. For an unconditionally stable circuit, K > 1 for all frequencies and  $|\Delta| < 1$ .

With this design, the amplifier is potentially unstable even though the stability circle lay outside of the Smith chart at the design frequency. The designer would then need to design a new bias circuit or select a different output resistor value.



Fig. 2. Stability circle shown in red for device at the design frequency. The circle lies outside the Smith chart, representing stability at the design frequency.



Fig. 3. Stability circle shown in red for device at the design frequency with a 600  $\Omega$  resistor at the output. The circle lies significantly outside the Smith chart, representing stability at the design frequency.



Fig. 4. Completed design schematic with 300  $\Omega$  shunt output resistor for stability



Fig. 5. Stability factor plot for the amplifier design with a 600  $\Omega$  resistor at the output. Since the stability factor is not greater than 1 for all frequencies, the device is potentially unstable.

# IV. SMITH TUBE DESIGN METHOD RESULTS

Using the three-dimensional Smith tube would help the designer select the correct parameters earlier in the design process. Usually, instability occurs at frequencies other than the design frequency. The Smith Tube stability surface presents the designer with a complete picture of stability across a range of frequencies. For an unconditionally stable device, the stability surface must lie outside of the Smith tube at all frequencies. Figure 6 shows the Smith tube stability surfaces for the device with no output resistor, a 600  $\Omega$  resistor, and a 300  $\Omega$  resistor, respectively.

Using the Smith tube as a design tool, the designer can see that the 300  $\Omega$  resistor at the output will achieve unconditional stability. This is confirmed by simulation results. Figure 7 shows the plot of the stability factor for the same schematic shown in figure three but with a 300

 $\Omega$  resistor at the output. The plot shows that the circuit meets the *K* criterion for unconditional stability. In this case, the designer saved time by selecting an output resistor value using knowledge of the stability surface over a range of frequencies rather than at just the design frequency.

This visualization tool allows designers to see clearly in one picture the variation of stability with frequency, and to adjust designs accordingly. For example, future developments could include plotting the trajectory of the matching network reflection coefficient over frequency alongside the stability circle. This would ensure that the design remains in the stable region at all frequencies of operation. The Frequency Smith Tube facilitates more efficient and accurate circuit design methods.

#### V. CONCLUSIONS

Compared to the conventional method for amplifier design, the Frequency Smith Tube method saves time and frustration. The Frequency Smith Tube provides a way to visualize the amplifier's stability and other parameters over frequency. If the stability surface is a safe distance from the Smith Tube, the amplifier will be unconditionally stable. The Frequency Smith Tube can be extended to many bandwidth related design problems. The Smith Tube can also illustrate changes in Sparameters, gain circles, noise performance, and other parameters over frequency.



Fig. 6. Stability surface plotted for the device with (a) no resistor at the output, (b)  $600\Omega$  shunt resistor at the output, and (c)  $300 \Omega$  shunt resistor at the output. The device is potentially unstable in (a) and (b), but unconditionally stable in (c).



Fig. 7. Stability factor plot for the amplifier design with a 300  $\Omega$  resistor at the output. Since the stability factor is greater than 1 for all frequencies, the device is unconditionally stable.

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